

Competition for Vilnius university students

2006

1. Let n be a positive integer exceeding 1. How many permutations $\{a_1, a_2, \dots, a_n\}$ of $\{1, 2, \dots, n\}$ are there which maximize the value of the sum

$$|a_2 - a_1| + |a_3 - a_2| + \dots + |a_{i+1} - a_i| + \dots + |a_n - a_{n-1}|$$

over all permutations? What is the value of this maximum sum?

2. How many $n \times n$ invertible matrices A are there for which all the entries of both A and A^{-1} are either 0 or 1?
3. Let a be a nonzero real and u and v be real 3-vectors. Solve the equation

$$2ax + (v \times x) + u = 0$$

for the vector x .

4. Prove that

$$\int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} + \dots$$