Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University

2016-02-06

(organized by Paulius Drungilas, Artūras Dubickas and Jonas Jankauskas)

Problem 1. Let S be the set of all real numbers x satisfying $x^4 + 36 \leqslant 13x^2$. Find the largest and the smallest values of $x^3 - 3x$ when $x \in S$.

Problem 2. Prove that for every matrix $A \in M_2(\mathbb{R})$ there exist two matrices $B, C \in M_2(\mathbb{R})$ such that

$$A = B^3 + C^3.$$

(Here, $M_2(\mathbb{R})$ denotes the set of 2×2 matrices with real coefficients.)

Problem 3. Let

$$P(x) = x^{2016} + a_1 x^{2015} + a_2 x^{2014} + \cdots + a_{2015} x + a_{2016}$$

be a polynomial of degree 2016 whose coefficients a_j belong to the set $\{-1, 1\}$ for each $j = 1, 2, \dots, 2016$. Prove that P has less than 2016 distinct real roots.

Problem 4. Find the value of the limit

$$\lim_{n\to\infty} \sum_{i=0}^{n} \sum_{j=0}^{n} \frac{1}{(i+j+1)i!j!}.$$

Each problem is worth 10 points.