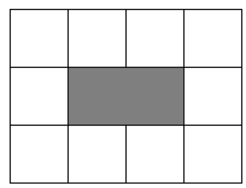
Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University

2018-02-17

(organized by Paulius Drungilas and Artūras Dubickas)

Problem 1. Is it possible to place the integers from 1 to 10 in the unshaded boxes of



the table in such a way that the four sums of numbers in two rows of four boxes and in two columns of three boxes

a) are all equal to 20?
b) are all equal to 16?

Problem 2. Is it true that for each positive integer n there exists a positive integer m such that $n|m$ and the sum of the digits of m equals n?

Problem 3. Find all pairs of positive integers (m, n) for which there exists a polynomial with real coefficients $P(x, y)$ satisfying the following four conditions

(1) $\deg_x P = m$,
(2) $\deg_y P = n$,
(3) $P(x, y) > 0$ for all $(x, y) \in \mathbb{R}^2$,
(4) $\inf_{(x,y)\in\mathbb{R}^2} P(x, y) = 0$

or prove that there are no such pairs (m, n).

Problem 4. Let $f : \mathbb{R} \to \mathbb{R}$ be a non-decreasing function satisfying $f(1) > 2$ and $f(2) < 3$. Prove that $f(x) = x + 1$ for some $x \in (1, 2)$.

Each problem is worth 10 points.