Mathematical Competition for Students (MIFMO)
of the Department of Mathematics and Informatics
of Vilnius University

2024-02-10

(organized by Paulius Drungilas and Artūras Dubickas)

Problem 1. As a monkey was carrying three coconuts to the top of a multistory building, one of the nuts fell to the ground from the 16th floor and crashed. The monkey decided to determine the lowest floor (from the first to 16th) for the nut to crash when dropped to the ground. The monkey still has two coconuts. Any of those two can be dropped from any of the floors, and then picked up and used again for a new trial if it did not crash. Is it true that at most five trials are enough for the monkey to determine the lowest floor from which a coconut crashes?

Problem 2. Let $n > m$ be positive integers. Suppose that a complex number z_0, with $|z_0| = 1$, is a root of the polynomial $z^n - z^m + 1$. Prove that z_0 is a root of unity, i.e., there exists a positive integer N such that $z_0^N = 1$.

Problem 3. Find all functions $f : (0, +\infty) \to (0, +\infty)$ satisfying

$$f^2(x) \geqslant f(x + y)(f(x) + y)$$

for all $x, y > 0$.

Problem 4. Do there exist positive integers $a_1 \leqslant a_2 \leqslant \ldots \leqslant a_{101}$ such that

$$a_1 a_2 \ldots a_{101} = \sum_{1 \leqslant i < j \leqslant 101} \operatorname{lcm}(a_i, a_j),$$

where $\operatorname{lcm}(a, b)$ stands for the least common multiple of positive integers a and b?

Each problem is worth 10 points.