Mathematical Competition for Students (MIFMO) of the Department of Mathematics and Informatics of Vilnius University

2026-02-07

(organized by Paulius Drungilas and Artūras Dubickas)

Problem 1. Find the smallest positive integer k with the following property: there exist five distinct integers m_1, m_2, m_3, m_4, m_5 such that the polynomial

$$P(x) = (x - m_1)(x - m_2)(x - m_3)(x - m_4)(x - m_5)$$

has exactly k nonzero coefficients.

Problem 2. Let $\mathcal{S}$ be the set of all positive integers that can be written as

$$\frac{1}{a_1} + \frac{2}{a_2} + \cdots + \frac{10}{a_{10}},$$

where $a_1, a_2, \ldots, a_{10}$ are (not necessarily distinct) positive integers.

(i) Prove that $1 \in \mathcal{S}$.

(ii) Prove that $12 \in \mathcal{S}$.

(iii) Find the largest element of $\mathcal{S}$.

(iv) Find the set $\mathcal{S}$.

Problem 3. Show that for each $a > 0$ the integral

$$\int_0^{\pi/2} \frac{(\cos x)^a}{(\sin x)^a + (\cos x)^a} dx$$

is convergent and find its value.

Problem 4. Let A and B be 2×2 matrices with integer entries such that A, $A + B$, $A + 2B$, $A + 3B$, and $A + 4B$ are all invertible matrices whose inverses have integer entries. Show that for each $t \in \mathbb{Z}$ the matrix $A + tB$ is invertible and that its inverse has integer entries.

Each problem is worth 10 points.